FCC/OCE RS79-01

INVESTIGATION OF METHODS FOR CONVERTING THE FCC GROUND WAVE FIELD INTENSITY CURVES TO THE METRIC SYSTEM

BY JOHN H. McMAHON
RESEARCH & STANDARDS DIVISION
OFFICE OF CHIEF ENGINEER

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Abstract

This report describes various methods given in the literature for calculation of ground wave field intensities in the Standard Broadcast band. This examination was made for the purpose of developing a computer program which could be used to revise the present FCC Ground Field Intensity Curves to the metric system. No method of calculation was found which exactly duplicated field intensities given in the existing FCC curves. The report contains four figures comparing field intensities predicted by several methods and by existing FCC curves. Also included in the report is a listing of the computer programs developed during the study for calculation of field intensities.

INTRODUCTION

It appears desirable in keeping with United States Government directives to replace English units and, accordingly, to revise the FCC Ground Wave Propagation Curves contained in Part 73 of the Commission's Rules to show distances in kilometers rather than miles. This conversion appears trivial since it would seem only necessary to divide the mileages by 1.609 and change the distance scale to kilometers. However, to be useful, major graphical divisions should be assigned to integral numbers of kilometers to simplify interpolation by the users. Accordingly, this revision requires at a minimum, a re-drafting of these curves. If it is desired only to revise the distance scales, an inverse field of 160.9 Millivolts per meter should be used. If it is deemed necessary to use 100 Millivolts per meter at 1 kilometer, complete re-drafting of these curves is required.

The FCC Broadcast Bureau has pointed out that the present Ground Wave curves contain some original drafting errors which should be corrected during the metric revision. Since it also appears desirable to develop a computer program to calculate ground-wave field intensities in the Standard Broadcast band, the logical method to be used in the revision of these curves is to first develop the computer program. This program could be used then with available computer graphic programs to redraw the curves. This computer program would also be available for direct calculation of ground-wave field intensities.

An investigation has been made of the feasibility of developing a computer program for this purpose. Because it is not desired to perpetuate any errors in the present curves, two simple methods of developing such a program cannot be used. These methods are: 1. Develop a computer program which uses polynomial expressions fitted to the present curves and which contains the necessary logic to select the proper expression for the given frequency and ground constants. 2. Digitize the curves into sets of tables and devise table-look programs and interpolation programs to make the proper table selections for frequencies and ground constants. With these two methods disqualified, the programming method selected should use basic mathematical expressions relating field intensities to frequency, ground constants, and effective earth curvature.

A survey of the literature has been made, and three different computer programs incorporating different methods of calculating ground wave field intensities have been developed. Fields predicted by these programs have been compared to each other and to the fields given in the present FCC Ground Wave Field Intensity curves.

SURVEY OF METHODS FOR PREDICTING GROUND WAVE FIELD INTENSITIES

A bibliography is given at the end of this report listing publications consulted in the preparation of this report. Bracketed numbers in the report refer to the same numbered reference in the bibliography. It appears that there is reasonably close agreement in the reference publications for prediction of ground wave field intensities for distances of about 100 km or less. For this distance range predictions are usually made using the Sommerfeld (1) surface wave field equations which were also treated by Norton (4), (5), and (7). Surface wave fields are predicted by use of the complex error function as discussed by Norton. At greater distances diffraction must be taken into account, and different calculation methods are required. For these greater distances, predictions are usually made using a residue series originally developed by Watson (2). Due to slow convergence of this residue series at smaller distances, there is a range of distances where neither the surface-wave equations nor the residue series, unless carried to a great number of terms, accurately predict ground wave field intensities.

Various authors have used different expedients to bridge this gap. Norton (5) suggests a method earlier proposed by Eckersley (6) to draw a continuous field intensity curve versus distance. The method consists in making two graphs of field intensity versus distance using identical scales of distances on the abcissae and field on the ordinate. One graph contains calculated surface-wave field intensities; the second, calculated residue-series field intensities. The graphs are superimposed on each other on a light table and the distance scales aligned. The graphs are moved vertically until the two curves are tangent to each other.

A composite curve is then prepared by using the surface-wave fields to the point of tangency and adjusted residue series fields beyond this point. The residue series adjustment consists in multiplying these field values by the ratio of the surface-wave field to the residue field at the tangent point.

Another method of preparing a composite ground-wave field-intensity curve has also been suggested by Norton (7). In this second method, Norton suggests that the surface-wave equations be used up to distances of 50/ FMhz 1/3 miles to prepare this part of the field intensity curve. At a considerable distance beyond the optical horizon a second curve is constructed using equations and graphs given in this reference. The graphs and equations relate field intensities to ground constants, frequency, and the radius of the earth as modified by diffraction. Between these two curves a smooth curve is drawn.

Bremmer (13) has suggested a third method of deriving a ground-wave field-intensity curve. He suggests that the Norton-Sommerfeld surface-wave field equations be modified to include a spherical-earth correction factor. (The Sommerfeld surface wave was derived for plane earth). The correction factor is a function of the effective earth radius, the wavelength, and earth constants. Bremmer states that with this correction the surface-wave fields will merge with the residue series fields approximately at the effective optical horizon. This merger may possibly occur for frequencies considerably above those for the Standard Broadcast band. However, tests of the Bremmer equations for Standard Broadcast frequencies with usual values of ground constants show that the residue fields are almost always smaller than the surface fields and do not merge with either the corrected, or uncorrected, surface-wave fields.

In addition to the fact that the surface-wave and residue-series curves do not merge, an even more serious problem, which has been encountered with the Bremmer spherical-earth correction to the surface-wave fields, is that the corrected surface-wave fields are erroneously large for high conductivities such as sea water at Standard Broadcast frequencies.

Bremmer does give considerable information on the residue series in this publication, and it is possible, if desired, to derive an unlimited number of residue-series terms from the equations given by Bremmer. Bremmer states that only three terms of the residue series are needed for most calculations. Al'Pert (11) used only two terms of this series in his field predictions. However, computer test programs written as part of this study show that at least six terms of the series are needed for convergence to 1 percent of the ultimate field value for distance of 100 kilometers at broadcast frequencies for the range of ground constants which may be encountered. More terms are needed at smaller distances. It has been found during the course of computer testing the Bremmer equations, that the uncorrected surface-wave field graphs parallel the residue-series field graphs for distances ranging from approximately from 40 to 110 kilometers. This parallelism makes a smooth merging of these curves possible by readjusting the ordinate values of the residue fields as suggested by Norton (5) with regard to merging of the surface-wave fields and Watson fields. If the ordinates of the residue series are not adjusted in this manner, a reverse "S" joining curve will be necessary to connect the two parallel primary curves. Both methods of joining the two fields have been tested. It appears that the method of readjusting the residue-series ordinates gives a smoother curve.

COMPUTER PROGRAMS DEVELOPED FOR GROUND WAVE FIELD INTENSITY PREDICTION

Three different computer programs have been developed to predict ground-wave field intensities using the three methods discussed in the previous section. The programs are built up from a number of subroutines chosen to perform the various calculations required. This type of program construction makes it easy to modify programs by changing only a particular subroutine without altering other parts of the program. Since all three programs calculate fields within the optical horizon using the Norton-Sommerfeld equations, only one set of subroutines is used by all three programs to calculate fields within the horizon distance. Beyond the optical horizon, the three programs differ in computation methods. Two of the programs, called respectively the Watson-Sommerfeld and the Bremmer, use different versions of the Watson residue series to calculate fields. The other program called the Norton 1941 uses a different set of formulas for this purpose.

Ultimately only one ground wave field prediction program is needed. However, during program development it has been found useful to have three different programs for cross-checking purposes. Fields predicted by these programs have also been compared against fields given in the FCC Ground Wave Propagation Curves. Figures 1-4 compare predictions by these four sources at .55 and 1.6 MHz for conductivities of 2 millimhos per meter and 5000 millimhos per meter and relative dielectric constants of 15 and 80 respectively. Except for Figure 3, these figures show very close agreement between the fields taken from the FCC Ground Wave Field Intensity Curves and those predicted by the Norton-Sommerfeld surface-wave computer program at distances within the effective horizon distance. Figure 3 shows the Norton-Sommerfeld fields falling several percent below the FCC curve values for distances within the horizon. It is believed that this difference is caused by the fact that the Norton-Sommerfeld fields in this figure were calculated for a frequency of 1600 kHz whereas the FCC fields which were taken from the FCC 1520-1600 kHz curves were probably calculated for a frequency of 1560 kHz, the mid frequency for this set.

Comparison of the fields shown for a conductivity of 2 millimhos, within horizon distances as given by the FCC 1430-1510 kHz curve and the FCC 1520-1600 kHz curve, show that these fields are approximately proportional to the inverse frequency ratio squared. On this basis, it is to be expected on Figure 3, that the Norton-Sommerfeld calculated fields would be about 5 percent less than the FCC fields within the horizon. This difference between the calculated fields and the FCC fields does not appear on Figure 4 because for sea water both fields are so near to the inverse distance field that there is very little effect from a variation of frequency.

The close agreement for within horizon distances between the FCC fields and the Norton-Sommerfeld fields shown in Figures 1 and 2 probably results from the fact that both curves were calculated for the same frequency of 550 kHz. The trends shown in these four figures have also been found to apply generally in numerous other comparisons. That is, for distances within the effective horizon, there is good agreement between FCC field predictions and those generated by the computer programs. The agreement among these four sets of fields is poorer over the horizon, and none of the computer programs gives fields which consistently agree with the FCC fields.

DESCRIPTION OF COMPUTER PROGRAM LISTING

Included in this report is a listing of the Fortran programs used during this study for calculation of ground wave field intensities. Although a MAIN program is included in the listing, various other versions of the MAIN program are possible depending upon which sets of calculations are desired. The function of the MAIN program is to merely introduce the variables---frequency, ground constants, and distances---to the various subroutines and then write out the calculated results in some desired format.

No large effort has been expended in the program development to choose the most efficient calculation methods, and undoubtly, some reduction in calculation times can be made by rearrangement of some of the subroutines. For example, many parameters which are functions of frequency and ground constants alone, but not of distance are calculated in the subroutines. It is therefore efficient to calculate these parameters once for a large set of distances. BREMR and SBREMR illustrate this arrangement by only calculating the residue series coefficients when frequency or ground constants are changed. A similar bypass for redundant calculations could probably be incorporated in subroutine ABG.

The three methods of calculating ground-wave field intensities used in this study as stated previously are called Norton 1941, Bremmer, and Watson-Sommerfeld respectively to indicate the source of each program. All three programs calculate fields within the optical horizon using the same set of subroutines NUMD and FPB. The three differ in calculating over-the-horizon fields. NUMD calculates P, the numerical distance, and B, the phase angle of P, from the ground constants, the frequency, and the distance.

Subroutine FPB then uses P and B from NUMD to calculate A, the surface wave attenuation factor below inverse distance fields, and PHI, the phase angle of A. FPB uses three different methods to calculate A depending upon the value of P. The methods were chosen to reduce calculation times by minimizing unnecessary iterations. For values of P equal to, or less than .7, P is calculated by use of the W function (14, p. 297). For values of P between .7 and 5, FPB uses a convergent infinite P series given by Norton (7, p. 637). For P values exceeding 5, A is calculated in a closed form using an equation for the W series (14, p. 328).

Taking the three methods in order, the Norton 1941 program calculates over-the-horizon fields using the subroutines FMVMA and ABG. FMVMA constructs a transition curve beyond the fields within the optical horizon and the fields at points well over the horizon using NUMD, FPB, and ABG. ABG calculates fields over the horizon using equations given by Norton in reference (7).

The second program called Bremmer uses residue series equations given by Bremmer (13) to calculate over-the-horizon fields. It uses subroutines FMVMB, BREMR, and SBREMR for these calculations. It has been found by experiment that the Bremmer residue fields lie in curves that parallel the surface wave fields curves for an appreciable distance on either side of the horizons. FMVMB and the MAIN program use this parallelism to join these curves at the horizon by multiplying the residue series fields by the ratio of the surface wave field to the residue field at the horizon.

The third program called the Watson-Sommerfeld program is based on formulas and tabular data given by Norton (4).

The Watson-Sommerfeld program calculates over-the-horizon fields using the Watson diffraction formula corrected for refraction in the lower atmosphere as suggested by Burrows (9), and incorporating a correction by Eckersley (6) for finite earth conductivity. Fields given by the Watson diffraction formula are less than the surface wave fields at equal distances. Consequently to join these two fields into a continuous curve it is necessary to move the Watson curve vertically until is is tangent to the surface wave curve. Watson fields are calculated by subroutines FOH and SFOH based on Norton (4). The joining of the two curves, which was formerly accomplished mechanically be superimposing graphs of surface-wave fields and Watson fields and then moving the Watson graph vertically for tangency, is now performed by subroutine MERGE. MERGE finds the distance at which the slopes of the two field curves are most nearly equal and sets a multiplier of the Watson field to produce equality of the two fields at this distance. Fields are calculated for distances beyond the tangent distance using the adjusted Watson equations. The surface wave equations are used up to the tangent distance.

Equations and tabular data on which the various subroutines are based are given in detail in the various references and in the interest of brevity are not repeated here, since the equations and tables are also given in the Fortran listing of these programs which is included in this report.

DISCUSSION AND RECOMMENDATIONS

During this study numerous comparisons have been made of the fields predicted by the listed computer programs and the fields predicted by the FCC Ground Wave Field Intensity curves. These comparisons have shown the same general trends evident in Figures 1-4 of this report, that is, there is good agreement between the computer program field predictions and the FCC curve fields out to the optical horizon at distances between 80 and 100 kilometers. At greater distances, field predictions of the computer programs and the FCC curves differ appreciably. A major influence on this difference appears to be the ground conductivity. For example, for sea water the Norton 1941 program predicts fields which are nearer to FCC curve values than those of the other two programs. For this conductivity the Norton 1941 program predictions are usually within 25 percent, or less, of the FCC curve fields. For lower ground conductivities, the field predictions of the other two programs more nearly approach the FCC curve fields than does the Norton 1941 predicted fields. The Bremmer program usually predicts fields which are about 10 percent above the FCC fields for distances of 100 to 200 kilometers, and then at greater distances falls more and more below the FCC fields to a value about 50 percent lower than the FCC fields at 1000 kilometers. The Watson-Sommerfeld program fields have an opposite slope to that of the Bremmer fields having roughly the same positive error difference in the 100 to 200 kilometer range and then the Watson-Sommerfeld fields continue to increase with distance above the FCC fields to a value about 50 percent above the FCC fields at 1000 kilometers.

It should be noted in passing that, which, if any, of these four sources of fields predictions is correct is not known. The region where the major differences of predictions lies, from 100 to 1000 kilometers, is a region where very few sets of field measurements have been taken. There are many reasons for the lack of measured ground wave field data in this region. Among these reasons are the following: (1) Field measurements for most broadcast station proofs of performance are made at distances which are less than 100 kilometers, (2) Except for all-sea-water paths, most other paths for distance of 100 kilometers, or more, will usually cross several regions of different conductivities making it difficult to relate ground constants and fields, (3) Only around mid-day are skywave signals not present over most of this range complicating determination of the true ground wave field values, (4) Interference from co-channel and adjacent signals may contaminate the desired ground wave signal, (5) At the larger distances man-made and atmospheric noise may have magnitudes which are comparable to the ground wave signal.

In view of the previous discussion which shows that, first, there is not consistent agreement between any of the computer predicted fields and the FCC curve fields, and second, that it is unlikely that field measurements can be made to find which method of predictions is correct, there appears to be no obvious choice of a method which is consistently better than the other methods for revising the FCC Ground Wave Field Intensity Curves. It would appear that complications which may arise due to revision of these curves should be weighed against advantages which may be obtained from having a computer program which gives consistent and reproducible field values at all distance and ground conductivities.

Of the computer programs developed in this study it is believed that the Bremmer program has the best theoretical justification being based on the more recent theoretical work and providing sufficient terms of the residue series to provide any degree of convergence desired in over-the-horizon ground wave fields. Within the effective horizon there is no difference between the programs since they use the same subroutines for field calculations.

Accordingly, if it is desired to choose a computer program from the three listed programs for revision of the FCC Ground Wave Field Intensity Curves, the Bremmer program is recommended. It is further recommended that some additional effort be expended in finding methods to increase the calculation efficiency of the program by eliminating all redundant operations. Calculation of the residue series fields and the surface wave fields involves mathematical operations with complex numbers. It is possible that some increase in program calculation efficiency would result if the programs were changed to use the 1106 computer complex number routines rather than those presently incorporated in the subroutines. It is also suggested that the Bremmer program be used to prepare tabular arrays to be used in a table-look program to be developed which will have a faster execution time than the Bremmer program. Not considered in this report are two ground-wave field intensity programs in development by ITS and CCIR, since they were unavailable for use during this program. However, an interim version of the CCIR program was studied in this project. This program also gave fields at variance with the FCC curves. It is understood that final versions of these two programs may be available in 1979.

The present propagation programs do not provide several procedures which are often needed in ground wave propagation studies. Among these missing procedures are: (1) A capability, given a value of field intensity, to find the corresponding distance, and (2) A capability to find field intensities for ground wave paths with multiple conductivities. Neither of these procedures is considered difficult to implement with conventional computer programming. The first procedure probably could be implemented most efficiently if the present propagation programs were used to prepare tables of field intensity versus distance for the required values of ground constants and frequency. The distance for a given field intensity would then be found with a table-look-up subroutine which would enter the tables on the distance axis and look for the given field intensity. The subroutine would interpolate between tabular fields when the desired field was not a tabular value. This first capability could also be provided, somewhat less efficiently, with a subroutine which would use the present propagation programs. This subroutine would assume distances in an iteratively, convergent manner, checking the desired field and the field at the assumed distance and then modifying the distance so as to bring the two fields nearer together until the desired degree of convergence was obtained.

The second capability, to predict ground wave fields for paths with multiple conductivities, can easily be programmed using the previously discussed distance/field subroutine and the existing propagation programs. The subroutine would use the present propagation program to find field at the first conductivity boundary. The subroutine would then use this field with the distance/field subroutine to find the equivalent distance for this field for the second conductivity. The subroutine would add the path length for the second conductivity to the equivalent distance and use the present propagation program to find the field at the next conductivity boundary. The subroutine would repeat this process as many times as required for the various conductivities.

ACKNOWLEDGMENT

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Acknowledgment is also given to William Daniel who read the report draft and made several helpful suggestions to improve the clarity of exposition of a number of technical details in the report.

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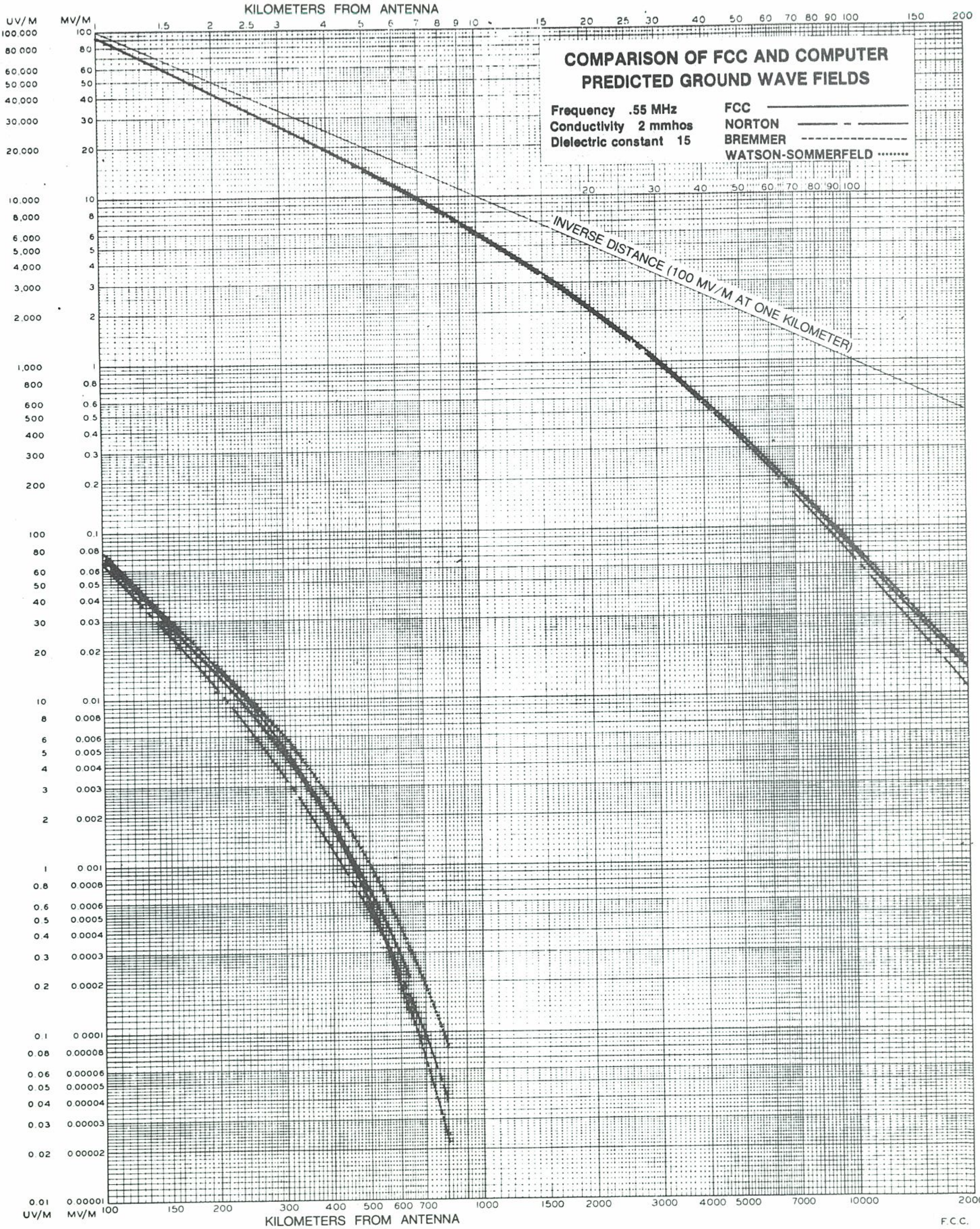


KILOMETERS FROM ANTENNA
COMPARISON OF FCC AND COMPUTER PREDICTED GROUND WAVE FIELDS
Frequency .55 MHz
Conductivity 2 mmhos
Dielectric constant 15
FCC
NORTON
BREMMER
WATSON-SOMMERFELD
INVERSE DISTANCE (100 MV/M AT ONE KILOMETER)
UV/M
MV/M
F.C.C.

Figure 1

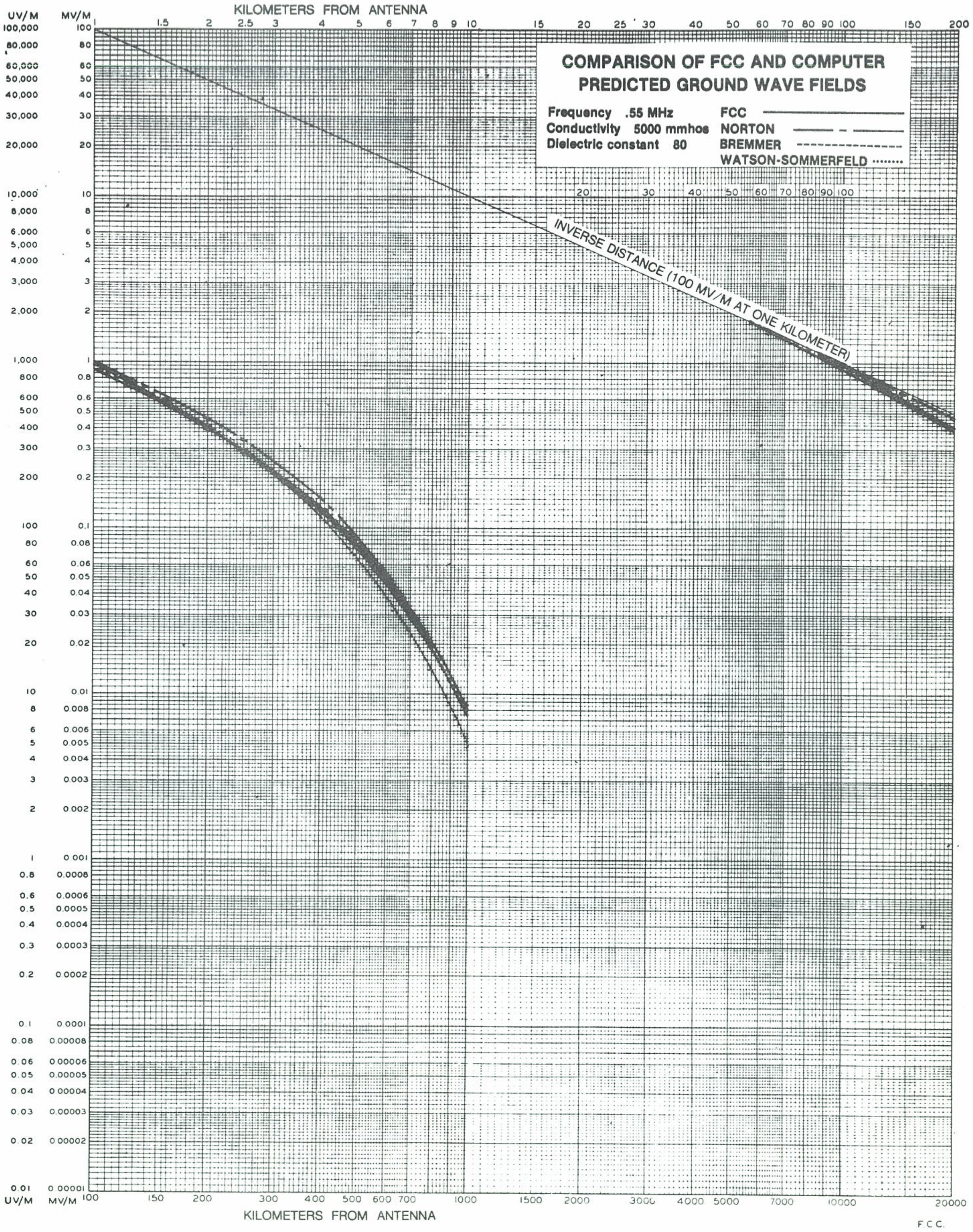
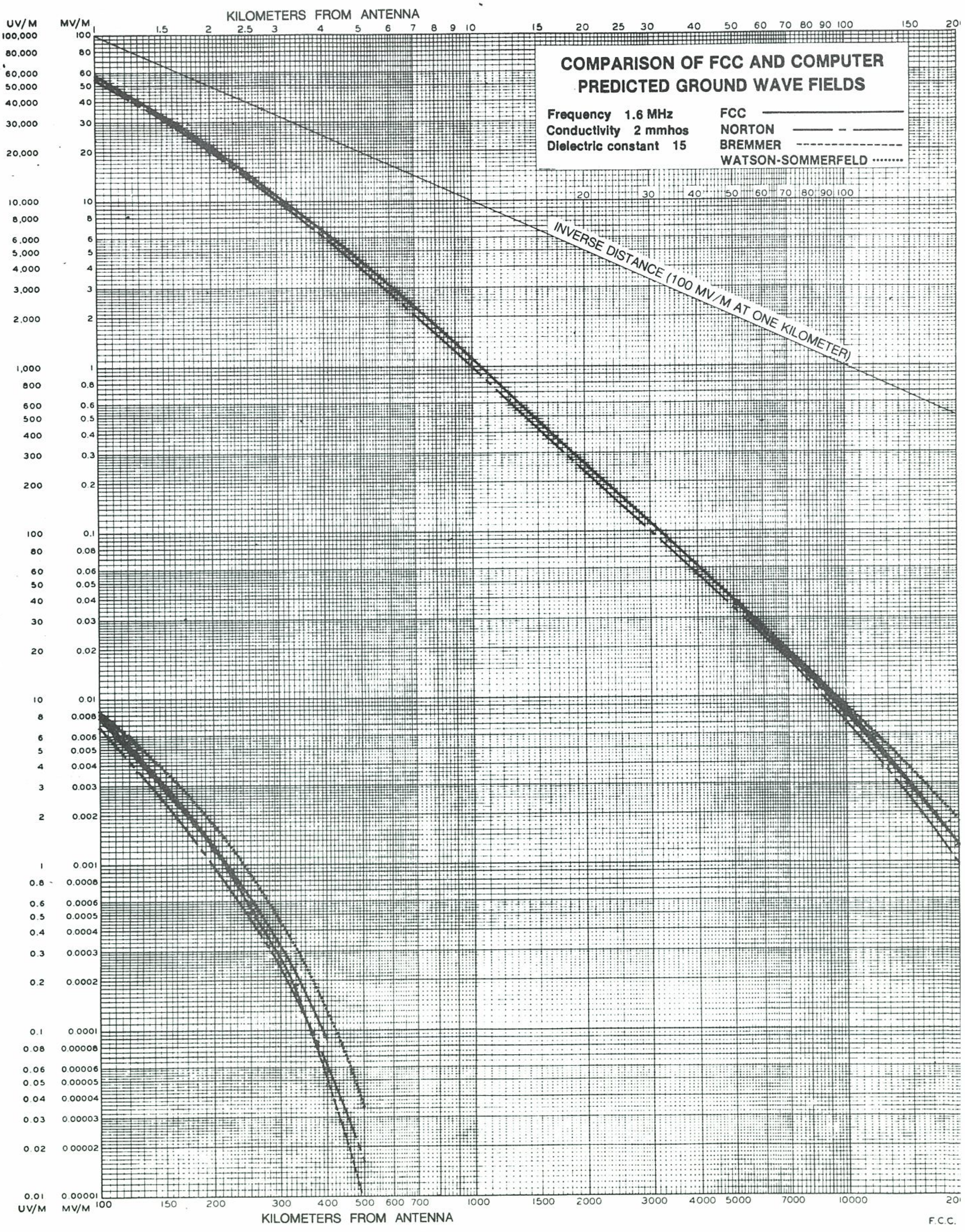
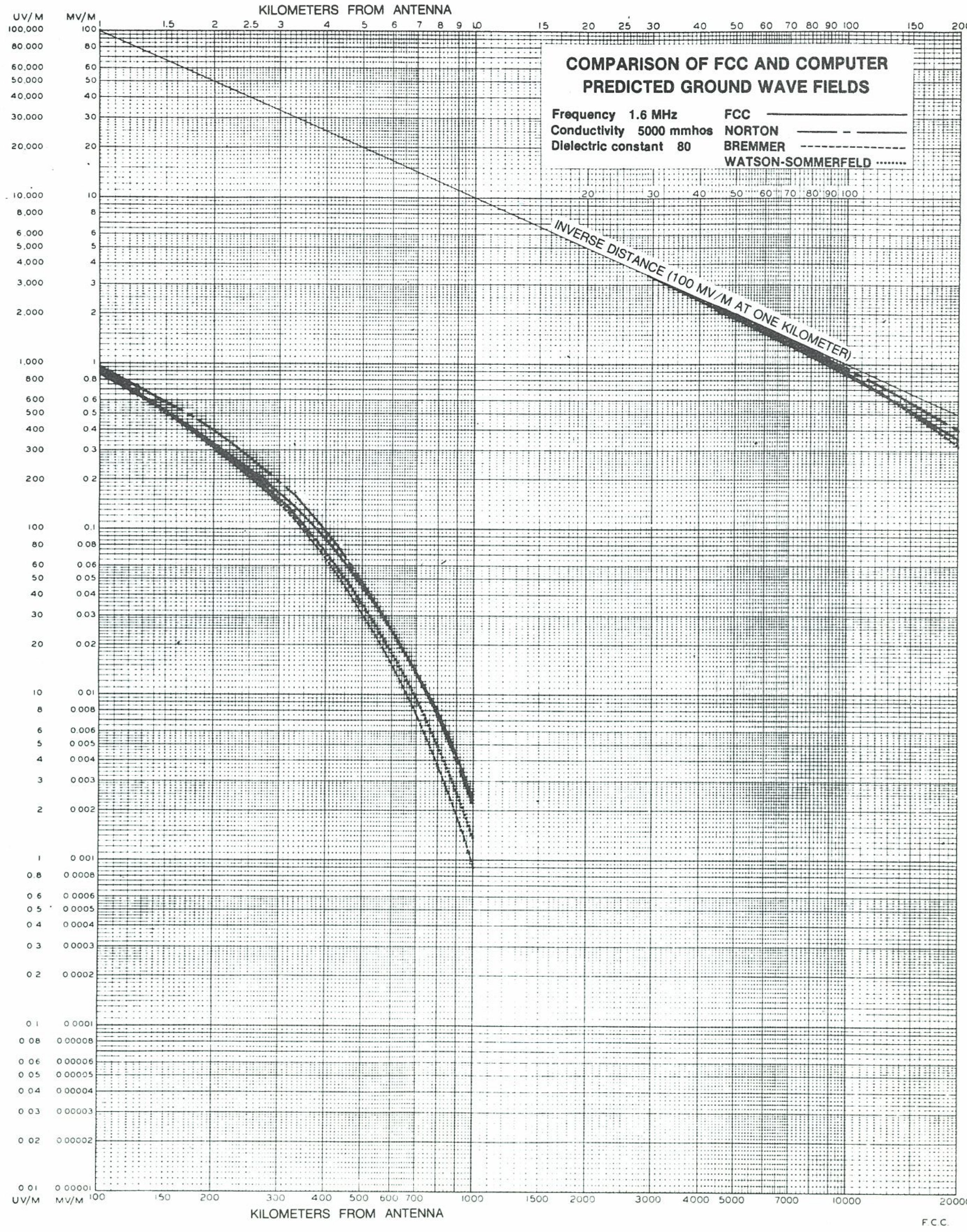

COMPARISON OF FCC AND COMPUTER
PREDICTED GROUND WAVE FIELDS
Frequency .55 MHz
Conductivity 5000 mmhos
Dielectric constant 80
FCC
NORTON
BREMMER
WATSON-SOMMERFELD
INVERSE DISTANCE (100 MV/M AT ONE KILOMETER)
KILOMETERS FROM ANTENNA
UV/M
MV/M
F.C.C.

Figure 2


COMPARISON OF FCC AND COMPUTER PREDICTED GROUND WAVE FIELDS
Frequency 1.6 MHz
Conductivity 2 mmhos
Dielectric constant 15
FCC
NORTON
BREMMER
WATSON-SOMMERFELD
INVERSE DISTANCE (100 MV/M AT ONE KILOMETER)
KILOMETERS FROM ANTENNA
UV/M
MV/M
F.C.C.



KILOMETERS FROM ANTENNA
COMPARISON OF FCC AND COMPUTER PREDICTED GROUND WAVE FIELDS
Frequency 1.6 MHz
Conductivity 5000 mmhos
Dielectric constant 80
FCC
NORTON
BREMMER
WATSON-SOMMERFELD
INVERSE DISTANCE (100 MV/M AT ONE KILOMETER)
UV/M
MV/M
KILOMETERS FROM ANTENNA
F.C.C.

Figure 4

LISTING OF FORTRAN COMPUTER PROGRAMS USED IN GROUND WAVE FIELD INTENSITY CALCULATIONS

```
RS*GPROP.MAIN
C
C      THIS PROGRAM COMPARES GROUND WAVE FIELDS
C      PREDICTED BY FIVE DIFFERENT METHODS.
C
C      1.  FCC FIELD-TAKEN FROM FCC GROUND WAVE FIELD INTENSITY CURVES.
C
C      2.  CCIR FIELD- TAKEN FROM PRINT-OUT OF CCIR COMPUTER PROGRAM.
C
C      3.  NORTON 1941 FIELD- PREDICTED BY COMPUTER PROGRAM BASED ON
C      K.A. NORTON,PROC. OF IRE, VOL.29,NO.12,DEC.1941.
C
C      4.  BREMMER FIELD-PREDICTED BY A COMPUTER PROGRAM BASED UPON ,
C      METHODS DISCUSSED IN 'TERRESTIAL RADIO WAVES', BY H. BREMMER,
C      ELSEVIER PUBLISHING CO. 1949
C
C      5.  WATSON-SOMMERFELD FIELD- PREDICTED BY A COMPUTER PROGRAM
C      WHICH MERGES AT THE POINT OF BEST TANGENCY THE SOMMERFELD-NORTON
C      SURFACE WAVE CURVE WITH THE WATSON DIFFRACTION CURVE PER K.A. NORTON
C      PROC OF IRE,VOL. 24,NO. 10, OCT. 1936
C
C      ALL FIELDS ARE NORMALIZED TO AN INVERSE FIELD OF 160.9 MV/M AT ONE KM.
C
       COMMON/C4/ICT,ICS
       COMMON/R2/ABC
       DIMENSION FCC(9,31)
       DIMENSION CCIR(9,31)
       DATA(FCC(1,I),I=1,31)/147.,115.,90.,68.,54.,41.,32.,24.,18.,13.,9.
      X7,7.,4.85,3.35,2.4,1.4,.9,.56,.32,.21,.11,.065,.04,.024,.013,.0065
      X,.0029,.00115,.00035,0.,0./
       DATA(FCC(2,I),I=1,31)/160.,127.,98.,77.,63.,46.,36.8,28.,23.,16.5,
      X13.,10.5,7.5,6.,4.5,3.2,2.4,1.6,1.1,.775,.45,.27,.16,.093,.054,.02
      X8,.013,.0055,.0019,.00055,.00012/
       DATA(FCC(3,I),I=1,31)/160.,129.,100.,80.,65.,49.,41.,33.,25.,20.,1
      X6.,13.,10.,8.,6.5,5.0,4.0,3.2,2.4,2.0,1.5,1.1,.85,.72,.46,.30,.2,.
      X12,.067,.032,.013/
       DATA(FCC(4,I),I=1,31)/120.,90.,68.,50.,36.,27.,19.5,13.7,9.3,6.25,
      X4.1,2.6,1.64,1.,.42,.35,.218,.13,.08,.054,.03,.0178,.0105,.0055,.0
      X028,.00125,.00032,.000155,0.,0.,0./
       DATA(FCC(5,I),I=1,31)/145.,116.,90.,69.,52.6,42.,32.,24.,17.9,13.1
      X,9.5,6.7,4.75,3.25,2.0,1.31,.82,.49,.29,.18,.096,.056,.033,.0194,.
      X0095,.0043,.0012,.0006,.000148,0.,0./
       DATA(FCC(6,I),I=1,31)/160.,128.,102.,80.,63.,51.,40.,31.9,25.2,20.
      X,16.,12.8,10.,8.1,6.4,5.,4.05,3.02,2.35,1.9,1.4,1.08,.8,.6,.41,.26
      X,.16,.094,.045,.0195,.0069/
       DATA(FCC(7,I),I=1,31)/90.,66.,47.,32.,23.,15.,10.5,6.7,4.3,2.7,1.7
      X,1.0,.62,.38,.24,.15,.09,.058,.035,.023,.013,.0075,.004,.002,.0009
      X5,.0004,.00014,0.,0.,0.,0./
       DATA(FCC(8,I),I=1,31)/135.,103.,79.,58.,44.,33.,24.,17.,12.,8.,5.5
      X,3.4,2.1,1.3,.76,.44,.26,.16,.095,.065,.035,.020,.011,.0056,.0027,
      X.0011,.0004,.00011,0.,0.,0./
       DATA(FCC(9,I),I=1,31)/160.,128.,100.,80.,65.,51.,40.,32.,25.,20.,1
      X6.,13.,10.,8.,6.5,4.9,3.9,3.0,2.3,1.9,1.4,1.05,.78,.54,.36,.23,.14
      X,.07,.03,.013,.0038/
       DATA(CCIR(1,I),I=1,31)/152.,131.,112.,94.3,78.6,64.7,52.3,41.5,32.
      X1,24.1,17.7,12.7,8.94,6.13,4.09,2.64,1.65,1.01,.596,.348,.201,.114
      X,.0646,.0355,.0187,.00918,.00409,.00158,.000504,.000124,.0000217/
```

```
      DATA(CCIR(2,I),I=1,31)/178.,154.,132.,113.,95.2,79.4,65.3,53.0,41.
     X9,32.8,25.4,19.5,14.9,11.2,8.28,6.03,4.31,3.00,2.03,1.32,.829,.498
     X,.285,.156,.0815,.0401,.0183,.00741,.00254,.000689,.000137/
      DATA(CCIR(3,I),I=1,31)/197.,171.,147.,125.,106.,88.3,72.8,58.6,47.
     X0,37.6,30.0,23.9,19.0,15.1,11.9,9.44,7.45,5.87,4.60,3.59,2.78,2.13
     X,1.60,1.18,.85,.587,.384,.233,.128,.0624,.0259/
      DATA(CCIR(4,I),I=1,31)/111.,94.,78.4,64.4,51.8,40.8,31.3,23.2,16.6
     X,11.5,7.51,4.80,2.99,1.83,1.11,.67,.404,.244,.146,.0873,.0505,.029
     X,.0161,.00853,.00420,.00187,.000721,.000229,.0000559,.00000975,.00
     X000111/
      DATA(CCIR(5,I),I=1,31)/170.,145.,123.,103.,85.5,69.6,55.6,43.6,33.
     X,24.5,17.9,12.8,8.9,6.03,3.96,2.51,1.53,.908,.526,.301,.171,.0953,
     X.0523,.0277,.0137,.00621,.00246,.000808,.000207,.0000382,.00000468
     X/
      DATA(CCIR(6,I),I=1,31)/211.,182.,155.,131.,110.,91.0,73.5,59.0,47.
     X2,37.7,30.0,23.9,18.9,15.0,11.9,9.38,7.38,5.79,4.52,3.50,2.68,2.03
     X,1.50,1.09,.755,.499,.307,.172,.0858,.0367,.013/
      DATA(CCIR(7,I),I=1,31)/65.9,54.7,44.7,35.8,27.9,21.2,15.5,10.9,7.3
     X5,4.71,2.93,1.81,1.12,.688,.423,.260,.159,.097,.0574,.0341,.0198,.
     X0112,.006,.00301,.00138,.000554,.000186,.0000488,.00000934,.000001
     X19,.0000000921/
      DATA(CCIR(8,I),I=1,31)/144.,121.,101.,82.2,65.8,51.5,39.3,29.0,20.
     X6,14.0,9.27,5.93,3.68,2.22,1.32,.778,.460,.273,.159,.0934,.0548,.0
     X302,.0163,.00823,.0038,.00155,.000528,.000142,.0000280,.00000373,.
     X000000302/
      DATA(CCIR(9,I),I=1,31)/222.,190.,162.,136.,113.,92.,73.9,59.2,47.3
     X,37.7,30.0,23.8,18.9,14.9,11.8,9.30,7.30,5.70,4.43,3.41,2.59,1.93,
     X1.40,.99,.666,.421,.245,.127,.0577,.0219,.00668/
      DIMENSION D(31)/1.00,1.26,1.58,2.00,2.51,3.16,3.98,5.01,6.31,7.94,
     X10.00,12.59,15.85,19.95,25.12,31.62,39.81,50.12,63.10,79.43,100.00
     X,125.89,158.49,199.53,251.19,316.23,398.11,501.19,630.96,794.33,10
     X00.00/
      DIMENSION SIG(3)/2.,6.,5000./
      DIMENSION E(3)/2*15.,80 ./
      DIMENSION FMHZ(3)/.55,1.0,1.6/
      DO 50 I=1,3
      DO 100 J=1,3
      WRITE(6,110) FMHZ(I),SIG(J),E(J)
  110 FORMAT(1H1,10X,'FREQUENCY, ',F4.2,' MHZ, CONDUCTIVITY, ',F5.0,' MI
     XLLIMHOS, DIELECTRIC CONSTANT, ',F5.0,'.',//,23X,'FIELDS ARE IN MIL
     XLIVOLTS PER METER',/,1X,'DISTANCE,KM',2X,'FCC FIELD',2X,'CCIR FIEL
     XD',2X,'NORTON 1941 FIELD',2X,'BREMMER FIELD',2X,'WATSON-SOMMERFELD
     X FIELD',//)
      ICT=0
      ICS=0
      CALL MERGE(SIG(J),E(J),FMHZ(I),ASC,ADIST)
      AM=ASC*.7944
      KZ=0
      DA=80.467/((FMHZ(I))**.33333)
      CALL NUMD(SIG(J),E(J),DA,FMHZ(I),P,B,DL)
      CALL FPB(P,B,A,PHI,N)
      EX=(160.93*A)/DA
      CALL BREMR(E(J),SIG(J),FMHZ(I),DA,EY)
      ABC=EX/EY
      L=(I-1)*3+J
      DO 150 K=1,31
      IF(D(K).GT.ADIST) GO TO 160
```

```
      CALL NUMD(SIG(J),E(J),D(K),FMHZ(I),P,B,DL)
      CALL FPB(P,B,A,PHI,N)
      EWAT=(160.9*A)/D(K)
      GO TO 170
  160 IF(KZ.EQ.1) GO TO 165
      CALL FOH(SIG(J),E(J),FMHZ(I),D(K),AM,BK,PZ1,A2)
      EWAT=(160.9*A2)/D(K)
      KZ=1
      GO TO 170
  165 CALL SFOH(FMHZ(I),D(K),BK,AM,A2)
      EWAT=(160.9*A2)/D(K)
  170 CALL FMVMA(SIG(J),E(J),FMHZ(I),D(K),EMV)
      ENOR=EMV
  180 IF(D(K).GT.D(1))ICT=1
      CALL FMVMB(SIG(J),E(J),FMHZ(I),D(K),EMV)
      EBRM=EMV
      CCIRA=CCIR(L,K)*160.9/173.
      WRITE(6,155)D(K),FCC(L,K),CCIRA,ENOR,EBRM,EWAT
  155 FORMAT(3X,F6.1,4X,F10.6,2X,F10.6,5X,F10.6,8X,F10.6,7X,F10.6)
  150 CONTINUE
      WRITE(6,215)
  215 FORMAT(///,5X,'EACH SET OF FIELDS NORMALIZED TO AN INVERSE DISTANC
     XE FIELD OF 160.9 MILLIVOLTS PER METER AT 1 KILOMETER.')
  100 CONTINUE
   50 CONTINUE
      END
```

```
RS*GPROP.FMVMA
      SUBROUTINE FMVMA(SIG,E,FR,D,EMV)
C
C     FMVMA PROVIDES FIELDS AT DISTANCES BETWEEN SURFACE WAVE CURVE
C     AND NORTON 1941 OVER HORIZON CURVE CALCULATED BY ABG BY
C     INTERPOLATING BETWEEN THE TWO CURVES.
C
      REAL NO
      REAL K
      DA=80.467/(FR**.333333)
      IF(D.GT.DA) GO TO 35
      CALL NUMD(SIG,E,D,FR,P,B,DL)
      CALL FPB(P,B,A,PHI,N)
      EMV=(A*160.93)/D
      RETURN
   35 CALL ABG(SIG,E,FR,D,E1,K,ALPHA,BETA,GAMMA,NO,DN2,ES2)
      DMIN=.75*DN2
      IF(D.LT.DMIN) GO TO 40
      EMV=E1
      RETURN
   40 CALL NUMD(SIG,E,D,FR,P,B,DL)
      CALL FPB(P,B,A,PHI,N)
      F1=(1.-((D-DA)/(DMIN-DA)))**2.5
      F2=1.-F1
      EMV=((A*160.93)/D)*F1+E1*F2
      RETURN
      END
```

```
RS*GPROP.FMVMB
          SUBROUTINE FMVMB(SIG,E,FR,D,EMV)
C
C         FMVMB AND MAIN NORMALIZE RESIDUE SERIES FIELDS TO
C         SURFACE WAVE FIELDS AT HORIZON. FPB IS USED FOR WITHIN
C         HORIZON FIELDS. NORMALIZED BREMR FIELDS ARE USED BEYOND
C         THE HORIZON
C
          COMMON/C4/ICT,ICS
          COMMON/R2/ABC
          DA=80.467/(FR**.333333)
          IF(D.GT.DA) GO TO 35
          CALL NUMD(SIG,E,D,FR,P,B,DL)
          CALL FPB(P,B,A,PHI,N)
          EMV=(A*160.93)/D
          RETURN
       35 IF(ICT.EQ.1.AND.ICS.EQ.1) GO TO 38
          CALL BREMR(E,SIG,FR,D,E1)
          ICS=1
          GO TO 39
       38 CALL SBREMR(D,E1)
       39 EMV=E1*ABC
          RETURN
          END
RS*GPROP.NUMB-SYM NOT FOUND
```

MCMAHON*TPF$.NUMD

```
      SUBROUTINE NUMD(SIG,E,D,F,P,B,DL)
C     NUMD CALCULATES NUMERICAL DISTANCE AND OTHER PARAMETERS
C     USED BY FPB TO CALCULATE SURFACE WAVE FIELDS.
C
C     SIG, GROUND CONDUCTIVITY IN MILLIMHOS PER METER
C     E,RELATIVE DIELECTRIC CONSTANT OF GROUND. AIR EQUALS UNITY.
C     D, KILOMETERS
C     F,FREQUENCY IN MHZ
C     P, NUMERICAL DISTANCE PER NORTON
C     B, PHASE ANGLE OF P IN DEGREES
C     DL, MAXIMUM DISTANCE IN KILOMETERS FOR F(P,B) FUNCTION VALIDITY.
C
      REAL LAM
C     THE FOLLOWING PROGRAM COMPARES GROUND WAVE FIELD INTENSITIES
      COMMON/F2/DEL,DELA
      X=17.9731*SIG/F
      B1=ATAN2((E-1.),X)
      B2=ATAN2(E,X)
      B=2.*B2-B1
      LAM=.299776/F
      P=(3.14159265*D*(COS(B2))**2)/(LAM*X*COS(B1))
      DL=80.46/(CBRT(F))
      DEL=(.01957*SQRT(E**2+X**2))/((((E-1.)**2+X**2)**.25)*F**.33333)
      PHI=ATAN2(E,X)-.5*ATAN2((E-1.),X)
      DELA=2.356-PHI
      RETURN
      END
```

RS*GPROP.FPB

```
      SUBROUTINE FPB(P,B,A,PHI,N)
C
C     FPB CALCULATES THE SURFACE WAVE ATTENUATION FACTOR A BY
C     ONE OF THREE METHODS DEPENDING UPON THE VALUE OF P THE
C     NUMERICAL DISTANCE. ITERATION IS CONTINUED UNTIL A CONVERGES
C     TO WITHIN 0.1 PERCENT OF FINAL VALUE
C
      COMMON/F2/DEL,DELA
      DIMENSION TEST(50)
      DIMENSION R(50),X(50)
      BR=B
      IF(P.GT..7) GO TO 5
      P2=SQRT(P)
      B2=B/2.
      GAMA=1.
      GAMO=1.7724538509
      REAL=1.
      AIMG=0.
      DO 100 I=1,50
      EX=FLOAT(I)
      ARC=(FLOAT(I))/2.
      IF(MOD(I,2).EQ.0) GO TO 210
      GAMO=GAMO*ARC
      GAM=GAMO
      GO TO 220
  210 GAMA=GAMA*ARC
      GAM=GAMA
  220 REAL=REAL+((P2**EX)/GAM)*COS(EX*(B2+1.57079633))
      AIMG=AIMG+((P2**EX)/GAM)*SIN(EX*(B2+1.57079633))
      TEST(I)=SQRT(REAL**2+AIMG**2)
      IC=I
      IF(I.EQ.1) GO TO 100
      IF((ABS((TEST(I))/(TEST(I-1))-1.0000)).LT..001) GO TO 110
  100 CONTINUE
      WRITE(6,108) P
  108 FORMAT(/,1X,'P EQUAL TO ',F6.3,' DID NOT CONVERGE.',/)
      RETURN
  110 AR=1.+1.77245385*P2*(TEST(IC))*COS(ATAN2(AIMG,REAL)+1.57079633+B2)
      AI=1.77245385*P2*(TEST(IC))*SIN(ATAN2(AIMG,REAL)+1.57079633+B2)
      A=SQRT(AR**2+AI**2)
      PHI=ATAN2(AI,AR)
      N=IC
      GO TO 30
    5 IF(P.GT.5.) GO TO 20
      EPR=(1./2.71828183**(P*COS(BR))*COS(P*SIN(BR)))
      EPI=-(1./2.71828183**(P*COS(BR))*SIN(P*SIN(BR)))
      REAL=1.+((SQRT(EPR**2+EPI**2))*(SQRT(3.14159265*P)))*COS(1.5707963
     X3+BR/2.+ATAN2(EPI,EPR))
      AIMG=((SQRT(EPR**2+EPI**2))*(SQRT(3.14159265*P)))*SIN(1.57079633+B
     XR/2.+ATAN2(EPI,EPR))
      IC=0
      FAC=1.
      P2=2.*P
      DO 10 I=1,101,2
      IC=IC+1
      AF=-1.
```

```
      IF(MOD(IC,2).EQ.0) AF=1.
      FAC=FAC*FLOAT(I)
      EXP=FLOAT(IC)
      ANG=BR*EXP
      FD=AF*P2**EXP
      REAL=REAL+(FD*COS(ANG))/FAC
      AIMG=AIMG+(FD*SIN(ANG))/FAC
      R(IC)=REAL
      X(IC)=AIMG
      TEST(IC)=SQRT(REAL**2+AIMG**2)
      IF(IC.EQ.1) GO TO 10
      IC1=IC-1
      IF((ABS(R(IC)/R(IC1)-1.0000)).LT. .001.AND.(ABS(X(IC)/X(IC1)-1.000
     X0)).LT. .001) GO TO 15
   10 CONTINUE
   11 WRITE(6,12)P,B
   12 FORMAT(1X,'P,',F6.2,' AND B',F6.2,' DID NOT CONVERGE IN 50 ITERATI
     XONS',/)
   15 A=TEST(IC)
      PHI=ATAN2(AIMG,REAL)
      N=IC
      GO TO 30
   20 RT1=SQRT(P**2-.380327*P*COS(BR)+.03616216)
      BT1=-ATAN2(P*SIN(BR),(P*COS(BR)-.1901635))
      RT2=SQRT(P**2-3.5689854*P*COS(BR)+3.1844142)
      BT2=-ATAN2(P*SIN(BR),(P*COS(BR)-1.7844927))
      RT3=SQRT(P**2-11.0506874 *P*COS(BR)+30.5293199)
      BT3=-ATAN2(P*SIN(BR),(P*COS(BR)-5.5253437))
      RE=(.4613135/RT1)*COS(BT1)+(.09999216/RT2)*COS(BT2)+(.002883894/RT
     X3)*COS(BT3)
      AI=(.4613135/RT1)*SIN(BT1)+(.09999216/RT2)*SIN(BT2)+(.002883894/RT
     X3)*SIN(BT3)
      RT=SQRT(RE**2+AI**2)
      BT=ATAN2(AI,RE)
      REAL=1.+1.77245385*RT*P*COS(BT+BR+3.14159265)
      AIMG=1.77245385*RT*P*SIN(BT+BR+3.14159265)
      A=SQRT(REAL**2+AIMG**2)
      PHI=ATAN2(AIMG,REAL)
      N=1
C     SPHERICAL EARTH CORRECTION PER BREMMER
   30 P2=SQRT(P)
      P1R=-.866*P2*(DEL**3)*COS(3.9268+2.*DELA)
      P1I=-.866*P2*(DEL**3)*SIN(3.9268+2.*DELA)
      P2R=1.62*P2*(DEL**4)*COS(2.880+3.*DELA)
      P2I=1.62*P2*(DEL**4)*SIN(2.880+3.*DELA)
      P3R=-.455*P2*(DEL**5)*COS(1.832+4.*DELA)
      P3I=-.455*P2*(DEL**5)*SIN(1.832+4.*DELA)
      P4R=-.304*P*P2*(DEL**5)*COS(.261+2.*DELA)
      P4I=-.304*P*P2*(DEL**5)*SIN(.261+2.*DELA)
      PTR=A*COS(PHI)+P1R+P2R+P3R+P4R
      PTI=A*SIN(PHI)+P1I+P2I+P3I+P4I
      A=SQRT(PTR**2+PTI**2)
      RETURN
      END
```

```
RS*GPROP.MERGE
            SUBROUTINE MERGE(SIG,E,F,ASC,ADIST)
      C
      C     MERGE FINDS THE POINT OF EQUAL SLOPE FOR FIELDS CALCULATED BY FPB
      C     (SURFACE WAVE) AND FOH( WATSON DIFFRACTION) AND ADJUSTS WATSON
      C     FIELDS FOR EQUALITY AT THIS POINT.(NORTON PROC. IRE OCT. 1936)
      C     MERGE USES NUMD AND FPB FOR SURFACE WAVE FIELD CALCULATIONS
      C     AND FOH AND SFOH TO CALCULATE WATSON RESIDUE SERIES FIELDS.
      C
            DIMENSION D(50),AH(50),AO(50)
            IT=0
            DO 50 I=10,500,10
            IT=IT+1
            D(IT)=FLOAT(I)
            CALL NUMD(SIG,E,D(IT),F,P,B,DL)
            CALL FPB(P,B,A,PHI,N)
            AH(IT)=A
            IF(I.GT.50) GO TO 45
            CALL FOH(SIG,E,F,D(IT),.7944,BK,PZ1,A2)
            AO(IT)=A2
            GO TO 50
         45 CALL SFOH(F,D(IT),BK,.7944,A2)
            AO(IT)=A2
         50 CONTINUE
            AMIN=100000000.
            DO 100 I=2,49
            DEL=ABS(((AH(I-1)-AH(I+1))/AH(I))-((AO(I-1)-AO(I+1))/AO(I)))
            IF(DEL .GT.AMIN) GO TO 100
            AMIN=DEL
            ASC=AH(I)/AO(I)
            ADIST=D(I)
        100 CONTINUE
            RETURN
            END

@PRT,S S.ABG,.FOH,.SFOH,.BREMR,.SBREMR
```

RS*GPROP.ABG

```
      SUBROUTINE ABG(SIG,E,FMHZ,D1,E1,K,ALPHA,BETA,GAMMA,NO,DN2,ES2)
C
C     ABG CALCULATES OVER THE HORIZON FIELDS USING EQUATIONS FROM NORTON
C     PROC. OF IRE VOL. 29,DEC. 1941
C
      REAL NO,K
      X=(17.9731*SIG)/FMHZ
      B1=ATAN2((E-1.),X)
      B2=ATAN2(E,X)
      B=2.*B2-B1
      K=(.0177737/FMHZ**.3333333)*SQRT((X*COS(B1))/((COS(B2))**2))
      IF(K.GT..7) GO TO 10
      ALPHA=.92787925-(K*COS(2.35619449-.5*B))-(1.2371723*K**3)*COS(1.83
     1259572-1.5*B)+(0.5*K**4)*COS(3.14159265-2.*B)-(2.7550715*K**5)*COS
     2(1.30899694-2.5*B)+(2.8867355*K**6)*COS(2.61799388-3.*B)+(6.589639
     3*K**7)*COS(3.92699082-3.5*B)+(13.3161793*K**8)*COS(2.0943951-4.*B)
      BETA=1.60713400- K*SIN(2.35619449-.5*B)-(1.2371723*K**3)*SIN(1.83
     1259572-1.5*B)+(0.5*K**4)*SIN(3.14159265-2.*B)-(2.7550715*K**5)*SIN
     2(1.30899694-2.5*B)+(2.8867355*K**6)*SIN(2.61799388-3.*B)+(6.589639
     3*K**7)*SIN(3.92699082-3.5*B)+(13.3161793*K**8)*SIN(2.0943951-4.*B)
      GO TO 20
   10 ALPHA=.40430826-(.61834008/K)*COS(.5*B-3.40339204)-(.2364189/K**2)
     1*COS(B-1.57079633)+(.053338757/K**3)*COS(1.5*B-2.87979327)-(.00225
     253514/K**4)*COS(2.*B-4.1887902)+(.0024693571/K**5)*COS(2.5*B-2.356
     319449)+(.00039940653/K**6)*COS(3.*B-.52359878)+(.000212015/K**7)*C
     4OS(3.5*B-1.83259572)-(.00046148795/K**8)*COS(4.*B)
      BETA=.700282450-(.61834008/K)*SIN(.5*B-3.40339204)-(.2364189/K**2)
     1*SIN(B-1.57079633)+(.053338757/K**3)*SIN(1.5*B-2.87979327)-(.00225
     253514/K**4)*SIN(2.*B-4.1887902)+(.0024693571/K**5)*SIN(2.5*B-2.356
     319449)+(.00039940653/K**6)*SIN(3.*B-.52359878)+(.000212015/K**7)*S
     4IN(3.5*B-1.83259572)-(.00046148795/K**8)*SIN(4.*B)
   20 GAMMA=.030046482*SQRT(BETA/(((ALPHA+(SIN(B))/(2.*K**2))**2)+((BETA
     X-(COS(B))/(2.*K**2))**2)))
      NO=.00358834*CBRT(FMHZ)
      DN2=2./(BETA*NO)
      ES2=160.9*NO*GAMMA
      AN=BETA*NO*D1
      E1=ES2*(56.6626/((2.71828183**(1.845527014*AN))*SQRT(AN)))
      RETURN
      END
```

```
RS*GPROP.FOH
      SUBROUTINE FOH(SIG,E,F,D,AM,BK,PZ1,A2)
C
C     FOH CALCULATES WATSON RESIDUE FIELDS USING DATA FROM NORTON,PROC.
C     OF IRE VOL 24 NO 10 OCT. 1936
C
      DIMENSION AK(37)/53.,53.,52.9,52.8,52.7,52.6,52.5,52.4,52.3,52.2,5
     12.1,52.0,51.9,51.8,51.6,51.3,50.8,49.7,48.5,46.8,43.2,39.3,34.8,29
     2.5,27.5,26.3,25.4,25.0,24.7,24.5,24.4,24.3,24.2,24.1,24.0,23.9,23.
     39/
      DIMENSION BA(37)/.0001,.0002, .0005,.001,.002,.005,.01,.02,.05,.1,
     1.2,.5,1.,2.,5.,10.,20.,50.,100.,200.,500.,1000.,2000.,5000.,10000.
     2,20000. ,50000.,100000.,200000.,500000.,1000000.,2000000.,5000000.
     3,10000000.,20000000.,50000000.,100000000./
      X=(17.9731*SIG)/F
      B1=ATAN2((E-1.),X)
      B2=ATAN2(E,X)
      B=2.*B2-B1
      BZ=(17.2*SIG)/(F**1.666667)
      DO 100 I=1,37
      IF(BZ-BA(I))105,108,100
  105 BK=AK(I-1)-(AK(I-1)-AK(I))*((BZ-BA(I-1))/(BA(I)-BA(I-1)))
      GO TO 110
  108 BK=AK(I)
      GO TO 110
  100 CONTINUE
  110 Z=.00038798*D*(F**.33333)*BK
      A2=0.
      EA=2.71828183
      AL=AM*SQRT(Z)
      POW=.5*(Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*EA**(-POW)
      POW=.5*(3.189*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.3135*EA**(-POW)
      POW=.5*(4.734*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.2110*EA**(-POW)
      POW=.5*(6.037*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.1653*EA**(-POW)
      POW=.5*(7.234*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.1382*EA**(-POW)
      POW=.5*(8.234*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.1199*EA**(-POW)
   20 CONTINUE
      RETURN
      END
```

RS*GPROP.SFOH

```
      SUBROUTINE SFOH(F,D,BK,AM,A2)
C
C     SINCE SFOH UTILIZES PARAMETERS PREVIOUSLY CALCULATED
C     BY FOH, IT HAS A FASTER EXECUTION TIME THAN DOES FOH
C     SFOH IS USED FOR REPEATED CALCULATIONS WHEN DISTANCE IS ONLY
C     PARAMETER CHANGED
C
      Z=.00038798*D*(F**.33333)*BK
      A2=0.
      EA=2.71828183
      AL=AM*SQRT(Z)
      POW=.5*(Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*EA**(-POW)
      POW=.5*(3.189*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.3135*EA**(-POW)
      POW=.5*(4.734*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.2110*EA**(-POW)
      POW=.5*(6.037*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.1653*EA**(-POW)
      POW=.5*(7.234*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.1382*EA**(-POW)
      POW=.5*(8.234*Z-1.)
      IF(POW.GT.89.) GO TO 20
      A2=A2+AL*.1199*EA**(-POW)
   20 CONTINUE
      RETURN
      END
```

RS*GPROP.BREMR

```
      SUBROUTINE BREMR(E,SIG,F,D,E1 )
C
C     BREMR CALCULATES THE OVER THE HORIZON FIELDS USING SIX TERMS
C     OF THE BREMMER RESIDUE SERIES PER 'TERRESTIAL WAVES' BY H.
C     BREMMER-1949
C
      COMMON/B5/AR,BR,CR,DR,ER,FR,AI,BI,CI,DI,EI,FI,ACHI,DELR,DELI,AK
      ACHI=.008024*F**.333333
      CHI=ACHI*D
      X=17.9731*SIG/F
      PHI=ATAN2(E,X)-.5*ATAN2((E-1.),X)
      AK=(.01957*SQRT(E**2+X**2))/((((E-1.)**2+X**2)**.25)*F**.33333)
      DELR=(1./AK**2)*COS(2.*(PHI-2.356))
      DELI=(1./AK**2)*SIN(2.*(PHI-2.356))
      IF(ABS(AKO-AK).LT..001) GO TO 20
      IF(AK.LT..7) GO TO 10
      AI=.6997-(.6188*SIN(.262-PHI))/AK+(.2369*COS(2.*PHI))/AK**2-(.0538
     X*SIN(.262+3.*PHI))/AK**3-(.00266*SIN(1.047-4.*PHI))/AK**4
      BI=2.232-(.194*SIN(.262-PHI))/AK+(.0073*COS(2.*PHI))/AK**2+(.01200
     X*SIN(.262+3.*PHI))/AK**3+(.0016*SIN(1.047-4.*PHI))/AK**4
      CI=3.312-(.1308*SIN(.262-PHI))/AK+(.00224*COS(2.*PHI))/AK**2-(.005
     X62*SIN(.262+3.*PHI))/AK**3-(.00034*SIN(1.047-4.*PHI))/AK**4
      DI=4.237-(.1022*SIN(.262-PHI))/AK+(.00107*COS(2.*PHI))/AK**2+(.003
     X46*SIN(.262+3.*PHI))/AK**3+(.000127*SIN(1.047-4.*PHI))/AK**4
      EI=5.067-(.0855*SIN(.262-PHI))/AK+(.00062*COS(2.*PHI))/AK**2-(.002
     X43*SIN(.262+3.*PHI))/AK**3-(.000062*SIN(1.047-4.*PHI))/AK**4
      FI=5.834-(.0742*SIN(.262-PHI))/AK+(.00041*COS(2.*PHI))/AK**2+(.001
     X83*SIN(.262+3.*PHI))/AK**3+(.000035*SIN(1.047-4.*PHI))/AK**4
      AR=.4040+(.6188*COS(.262-PHI))/AK-(.2369*SIN(2.*PHI))/AK**2-(.0538
     X*COS(.262+3.*PHI))/AK**3+(.00266*COS(1.047-4.*PHI))/AK**4
      BR=1.289+(.194*COS(.262-PHI))/AK-(.0073*SIN(2.*PHI))/AK**2+(.01200
     X*COS(.262+3.*PHI))/AK**3-(.0016*COS(1.047-4.*PHI))/AK**4
      CR=1.912+(.1308*COS(.262-PHI))/AK-(.00224*SIN(2.*PHI))/AK**2-(.005
     X62*COS(.262+3.*PHI))/AK**3+(.00034*COS(1.047-4.*PHI))/AK**4
      DR=2.446+(.1022*COS(.262-PHI))/AK-(.00107*SIN(2.*PHI))/AK**2+(.003
     X46*COS(.262+3.*PHI))/AK**3-(.000127*COS(1.047-4.*PHI))/AK**4
      ER=2.926+(.0855*COS(.262-PHI))/AK-(.00062*COS(2.*PHI))/AK**2-(.002
     X43*COS(.262+3.*PHI))/AK**3+(.000062*COS(1.047-4.*PHI))/AK**4
      FR=3.369+(.0742*COS(.262-PHI))/AK-(.00041*SIN(2.*PHI))/AK**2+(.001
     X83*COS(.262+3.*PHI))/AK**3-(.000035*COS(1.047-4.*PHI))/AK**4
      GO TO 20
   10 AI=1.607-AK*SIN(.785+PHI)-1.237*(AK**3)*SIN(1.309+3.*PHI)+.5*(AK**
     X4)*SIN(4.*PHI)-2.755*(AK**5)*SIN(1.309-5.*PHI)
      BI=2.810-AK*SIN(.785+PHI)-2.163*(AK**3)*SIN(1.309+3.*PHI)+.5*(AK**
     X4)*SIN(4.*PHI)-8.422*(AK**5)*SIN(1.309-5.*PHI)
      CI=3.795-AK*SIN(.785+PHI)-2.921*(AK**3)*SIN(1.309+3.*PHI)+.5*(AK**
     X4)*SIN(4.*PHI)-15.36*(AK**5)*SIN(1.309-5.*PHI)
      DI=4.664-AK*SIN(.785+PHI)-(3.590*AK**3)*SIN(1.3089+3.*PHI)+(.5*AK*
     X*4)*SIN(4.*PHI)-(23.21*AK**5)*SIN(1.3089-5.*PHI)
      EI=5.460-AK*SIN(.785+PHI)-(4.203*AK**3)*SIN(1.3089+3.*PHI)+(.5*AK*
     X*4)*SIN(4.*PHI)-(31.80*AK**5)*SIN(1.3089-5.*PHI)
      FI=6.202-AK*SIN(.785+PHI)-(4.774*AK**3)*SIN(1.3089+3.*PHI)-(.5*AK*
     X*4)*SIN(4.*PHI)-(41.02*AK**5)*SIN(1.3089-5.*PHI)
      AR=.928+AK*COS(.785+PHI)+1.237*(AK**3)*COS(1.309+3.*PHI)-.5*(AK**4
     X)*COS(4.*PHI)-2.755*(AK**5)*COS(1.309-5*PHI)
      BR=1.622+AK*COS(.785+PHI)+2.163*(AK**3)*COS(1.309+3.*PHI)-.5*(AK**
```

```
     X4)*COS(4.*PHI)-8.422*(AK**5)*COS(1.309-5.*PHI)
      CR=2.191+AK*COS(.785+PHI)+2.921*(AK**3)*COS(1.309+3.*PHI)-.5*(AK**
     X4)*COS(4.*PHI)-15.36*(AK**5)*COS(1.309-5.*PHI)
      DR=2.633+AK*COS(.785+PHI)+(3.590*AK**3)*COS(1.3089+3.*PHI)-(.5*AK*
     X*4)*COS(4.*PHI)-(23.21*AK**5)*COS(1.3089-5.*PHI)
      ER=3.153+AK*COS(.785+PHI)+(4.203*AK**3)*COS(1.3089+3.*PHI)-(.5*AK*
     X*4)*COS(4.*PHI)-(31.80*AK**5)*COS(1.3089-5.*PHI)
      FR=3.581+AK*COS(.785+PHI)+(4.774*AK**3)*COS(1.3089+3.*PHI)-(.5*AK*
     X*4)*COS(4.*PHI)-(41.02*AK**5)*COS(1.3089-5.*PHI)
   20 TR=0.
      TI=0.
      IF(ABS(AKO-AK).LT..001) GO TO 22
      E1R=2.*AR-DELR
      E1I=2.*AI-DELI
      E1T=SQRT(E1R**2+E1I**2)
      E1A=ATAN2(E1I,E1R)
      E2R=2.*BR-DELR
      E2I=2.*BI-DELR
      E2T=SQRT(E2R**2+E2I**2)
      E2A=ATAN2(E2I,E2R)
      E3R=2.*CR-DELR
      E3I=2.*CI-DELI
      E3T=SQRT(E3R**2+E3I**2)
      E3A=ATAN2(E3I,E3R)
      E4R=2.*DR-DELR
      E4I=2.*DI-DELI
      E4T=SQRT(E4R**2+E4I**2)
      E4A=ATAN2(E4I,E4R)
      E5R=2.*ER-DELR
      E5I=2.*EI-DELI
      E5T=SQRT(E5R**2+E5I**2)
      E5A=ATAN2(E5I,E5R)
      E6R=2.*FR-DELR
      E6I=2.*FI-DELI
      E6T=SQRT(E6R**2+E6I**2)
      E6A=ATAN2(E6I,E6R)
   22 D1=2.71828183**(-AI*CHI)
      A1=CHI*AR
      TR=TR+(D1/E1T)*COS(A1-E1A)
      TI=TI+(D1/E1T)*SIN(A1-E1A)
      D2=2.71828183**(-BI*CHI)
      A2=CHI*BR
      TR=TR+(D2/E2T)*COS(A2-E2A)
      TI=TI+(D2/E2T)*SIN(A2-E2A)
      D3=2.71828183**(-CI*CHI)
      A3=CHI*CR
      TR=TR+(D3/E3T)*COS(A3-E3A)
      TI=TI+(D3/E3T)*SIN(A3-E3A)
      D4=2.71828183**(-DI*CHI)
      A4=CHI*DR
      TR=TR+(D4/E4T)*COS(A4-E4A)
      TI=TI+(D4/E4T)*SIN(A4-E4A)
      D5=2.71828183**(-EI*CHI)
      A5=CHI*ER
      TR=TR+(D5/E5T)*COS(A5-E5A)
      TI=TI+(D5/E5T)*SIN(A5-E5A)
      D6=2.71828183**(-FI*CHI)
```

```
      A6=CHI*FR
      TR=TR+(D6/E6T)*COS(A6-E6A)
      TI=TI+(D6/E6T)*SIN(A6-E6A)
      FT=SQRT(TR**2+TI**2)
      E1 =(403.32/D)*(SQRT(CHI))*FT
      AKO=AK
      RETURN
      END
```

RS*GPROP.SBREMR

```
      SUBROUTINE SBREMR(D,E1 )
C
C     SBREMR USES THE RESIDUE COEFFICENTS CALCULATED BY BREMR TO
C     CALCULATE OVER THE HORIZON FIELDS WITHOUT REPEATING REDUNDANT,
C     CALCULATIONS IN BREMR.
C
      COMMON/B5/AR,BR,CR,DR,ER,FR,AI,BI,CI,DI,EI,FI,ACHI,DELR,DELI,AK
      CHI=ACHI*D
      TR=0.
      TI=0.
      IF(ABS(AKO-AK).LT..001) GO TO 22
      E1R=2.*AR-DELR
      E1I=2.*AI-DELI
      E1T=SQRT(E1R**2+E1I**2)
      E1A=ATAN2(E1I,E1R)
      E2R=2.*BR-DELR
      E2I=2.*BI-DELR
      E2T=SQRT(E2R**2+E2I**2)
      E2A=ATAN2(E2I,E2R)
      E3R=2.*CR-DELR
      E3I=2.*CI-DELI
      E3T=SQRT(E3R**2+E3I**2)
      E3A=ATAN2(E3I,E3R)
      E4R=2.*DR-DELR
      E4I=2.*DI-DELI
      E4T=SQRT(E4R**2+E4I**2)
      E4A=ATAN2(E4I,E4R)
      E5R=2.*ER-DELR
      E5I=2.*EI-DELI
      E5T=SQRT(E5R**2+E5I**2)
      E5A=ATAN2(E5I,E5R)
      E6R=2.*FR-DELR
      E6I=2.*FI-DELI
      E6T=SQRT(E6R**2+E6I**2)
      E6A=ATAN2(E6I,E6R)
   22 D1=2.71828183**(-AI*CHI)
      A1=CHI*AR
      TR=TR+(D1/E1T)*COS(A1-E1A)
      TI=TI+(D1/E1T)*SIN(A1-E1A)
      D2=2.71828183**(-BI*CHI)
      A2=CHI*BR
      TR=TR+(D2/E2T)*COS(A2-E2A)
      TI=TI+(D2/E2T)*SIN(A2-E2A)
      D3=2.71828183**(-CI*CHI)
      A3=CHI*CR
      TR=TR+(D3/E3T)*COS(A3-E3A)
      TI=TI+(D3/E3T)*SIN(A3-E3A)
      D4=2.71828183**(-DI*CHI)
      A4=CHI*DR
      TR=TR+(D4/E4T)*COS(A4-E4A)
      TI=TI+(D4/E4T)*SIN(A4-E4A)
      D5=2.1828183**(-EI*CHI)
      A5=CHI*ER
      TR=TR+(D5/E5T)*COS(A5-E5A)
      TI=TI+(D5/E5T)*SIN(A5-E5A)
      D6=2.1828183**(-FI*CHI)
```

```
      A6=CHI*FR
      TR=TR+(D6/E6T)*COS(A6-E6A)
      TI=TI+(D6/E6T)*SIN(A6-E6A)
      FT=SQRT(TR**2+TI**2)
      E1 =(403.32/D)*(SQRT(CHI))*FT
      AKO=AK
      RETURN
      END


@PRT,TL S.

RS*GPROP    ELEMENT TABLE

D  NAME          VERSION         TYPE          DATE       TIME       SEQ #  SIZE-PRE,TEXT  (CYCLE WORD) PSRMODE  LOCATION
   MAIN                          FOR SYMB      09 JAN 79  12:29:45    1            45      5    0    1              1792
   MAIN                          RELOCATABLE   09 JAN 79  12:29:56    2       2    67                               1837
   FMVMA                         FOR SYMB      09 JAN 79  12:30:00    3             6      5    0    1              1906
   FMVMA                         RELOCATABLE   09 JAN 79  12:30:10    4       1     8                               1912
   FMVMB                         FOR SYMB      09 JAN 79  12:30:14    5             5      5    0    1              1921
   FMVMB                         RELOCATABLE   09 JAN 79  12:30:18    6       2     6                               1926
   NUMD                          FOR SYMB      09 JAN 79  12:30:21    7             7      5    0    1              1934
   NUMD                          RELOCATABLE   09 JAN 79  12:30:25    8       2     7                               1941
   FPB                           FOR SYMB      09 JAN 79  12:30:28    9            26      5    0    1              1950
   FPB                           RELOCATABLE   09 JAN 79  12:30:39   10       2    39                               1976
   MERGE                         FOR SYMB      09 JAN 79  12:30:42   11             8      5    0    1              2017
   MERGE                         RELOCATABLE   09 JAN 79  12:30:46   12       1     7                               2025
   ABG                           FOR SYMB      09 JAN 79  12:30:49   13            14      5    0    1              2033
   ABG                           RELOCATABLE   09 JAN 79  12:31:01   14       2    30                               2047
   FOH                           FOR SYMB      09 JAN 79  12:31:07   15            12      5    0    1              2079
   FOH                           RELOCATABLE   09 JAN 79  12:31:18   16       1    18                               2091
   SFOH                          FOR SYMB      09 JAN 79  12:31:22   17             7      5    0    1              2110
   SFOH                          RELOCATABLE   09 JAN 79  12:31:30   18       1    10                               2117
   BREMR                         FOR SYMB      09 JAN 79  12:31:39   19            36      5    0    1              2128
   BREMR                         RELOCATABLE   09 JAN 79  12:31:51   20       2    56                               2164
   SBREMR                        FOR SYMB      09 JAN 79  12:31:54   21            14      5    0    1              2222
   SBREMR                        RELOCATABLE   09 JAN 79  12:32:00   22       2    19                               2236
   PRT             LIST          ELT SYMB      10 JAN 79  09:14:54   23             2      5    0    1              2257
NEXT AVAILABLE LOCATION-                                                                                            2259

ASSEMBLER PROCEDURE TABLE EMPTY

COBOL PROCEDURE TABLE EMPTY

FORTRAN PROCEDURE TABLE EMPTY

                          ENTRY POINT TABLE

 D NAME           LINK   D NAME          LINK   D NAME          LINK   D NAME          LINK   D NAME          LINK
   ABG              14     BREMR           20     FMVMA            4     FMVMB            6     FOH             16
   FORMAIN$          2     FPB             10     MERGE           12     NUMD             8     SBREMR          22
   SFOH             18


@BRKPT PRINT$
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